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Continuity and Steps: Musical and Mathematical Perspectives

Student Grade Level: 9th-10th

Class Subject: Math/Music

Class Size: N/A

Lesson Length: 45 minutes

Ohio Music Content Standards Addressed in this Lesson:

(Cross-referenced to the National Standards for Music Education)

Analyzing and Responding

- A. Describe and evaluate a piece of music using developed criteria based on elements of music and music vocabulary. (ns6)

Connections, Relationships, and Applications

- B. Demonstrate ways that subject matter of other disciplines is interrelated with that of music. (ns8)

Ohio Music Content Standards Addressed in this Lesson:

Patterns, Functions, and Algebra

- E. Analyze and compare functions and their graphs using attributes, such as rate of change, intercepts, and zeros.

Mathematical Processes

- B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

Mastery Objectives:

The students will be able to:

1. Describe continuous and stepwise functions using their mathematical definitions
2. Relate the concepts to stepwise and continuous functions to uses of pitch in Western and non-Western music

Materials:

1. A recording of a conventional western piece of music. In this case, Mozart's "Eine Kleine Nachtmusik" is used.
2. A recording on a nonwestern piece of music involving variable pitch and pitch bending. In this case, Gagaku music is used, specifically the recording "Gagaku - The Imperial Court Music of Japan" published by Lyrichord World.
3. Graphs illustrating stepwise and continuous functions. In this case, the functions $f(x) = |x|$ and $f(x) = x^2$ are used.

Prerequisites:

Students must understand what a function is and be familiar with the idea of Western scales and the twelve chromatic pitches in order to complete this lesson.

Procedures:

Anticipatory Set

- Have the students consider the terms "continuous" and "stepwise" as applicable in common English. What characteristics does something have if it is continuous? What characteristics does something have if it is stepwise?

Lesson Sequence/Instructional Strategies

- Play both recordings of the students and have them answer the following question: Considering pitch, which piece is continuous and which piece is stepwise?
- If the students have trouble, display for them the continuous and stepwise graph. Which matches better with which conception of pitch?
- Based on this experience, ask students to make an educated guess as to what a continuous function is in mathematics and how one can determine whether a function is continuous. This could function as a class discussion.
- The actual mathematical definition of a function in rigorous terms is difficult, but students might be able to point out observations such as the graph being smooth in the case of continuity and having breaks in the case of noncontinuity.
- Share with students a formal definition of a continuous function. (<http://mathworld.wolfram.com/> is a useful free reference for such definitions). Explain to them that this is an abstract way of stating what they had already discovered: the continuous function has a smooth graph with no breaks.
- Using this definition, ask the students how they might now describe the difference between the Mozart and the Gagaku music. The key is using formal language to describe the idea that Mozart has defined space between different pitches, measured roughly in semitones. Gagaku can have a smooth pitch slide in which every possible pitch between a starting and ending point is briefly sounded.

Closure

- With continuous pitch change, we cannot really measure how long each actual pitch lasts like we can with music with a finite number of defined pitches. How then, might students think to describe pitch change? One important answer, how fast the pitch is going up or down, both points toward physics and higher math and has a stark aesthetic affect on music.

Assessment:

- The students will complete problems that will evaluate and reinforce their grasp of the concepts. Tasks include evaluating mathematical functions for continuity (graphing the functions is recommended), and "graphing" musical excerpts as a way of studying how pitch is used. Examples should be of appropriate difficulty, i.e. not a function such as $f(x)$ such that $f(x) = 1$ if x is rational and $f(x) = 0$ if x is not rational. Two examples are given:
 1. Graph the function $f(x)$ such that $f(x) = 5$ if $x = 5$, otherwise $f(x) = 2x+1$. Where is this function continuous? Where is it not continuous?
 2. Listen to the opening flute solo of "Petrushka" and graph the musical shapes that you hear. Is the resulting graph continuous or stepwise? What does this say about the music?